Possible evidence for the mass shift of η' meson at finite temperature

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Abstract

Topological charge distributions of SU(3) gauge theory at finite temperature are calculated on the lattice with high statistics in a manner of free from the uncertainty of β-function. Clear temperature dependence of the topological charge distribution is obtained. Combining the present result with the recent study about the relation between the masses of η meson system and the gauge field topology, we obtain $m_{\eta'}(T)/m_{\eta'}(0) = 0.86 \pm 0.02$ and 0.69 ± 0.01 for $T = 0.75T_c$ and $T = 0.93T_c$, respectively. The result shows the first clear evidence of the mass shift of η' meson at finite temperature.

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1 Introduction

It is a common knowledge that at sufficiently high temperature, the hadronic system may undergo a phase transition to the QGP (quark-gluon-plasma) phase. What is a signature or a precursor of the QGP is a long standing question. Although a lot of possibilities have been posted so far, there is no definite answer yet [1]. Among them the mass shift of hadrons might be an promising candidate for the precursor of QGP if this phenomena is in the case. [2, 3, 4, 5]

This is because the mass shift, itself, can indicate that the system is very close to the critical region without a precise quantitative argument.

In this paper, we focus on the neutral pseudo-scalar meson system whose mass spectrum at zero temperature can be well explained by the linear sigma model incorporated with the axial U(1) anomaly [3]. It is also well known that the axial U(1) anomaly is caused by the topological fluctuations of the system,

$$\partial_\mu j_\mu(x) \propto Q(x), \tag{1}$$

where j_μ is the flavor singlet axial vector current and $Q(x)$ is the topological charge density.

Recently the relation between the singlet-octet mass difference of the neutral pseudo-scalar meson system and the topology has been analyzed in detail and the strong correlations of these two has been suggested [6].

From these results mentioned above, if there exist the temperature dependence in the topological sector of the theory, it might be

reflected by the possible change of the mass spectrum.

The organization of this paper is as follows. In the next section 2, we briefly mention our model of the neutral pseudo-scalar meson system as well as it's relation with the gauge field topology. The lattice results of the topological charge distributions at finite temperature is given in section 3. In the section 4, we will discuss the possible shift of the masses. The section 5 is devoted to the conclusion.

2 Model of the neutral pseudo-scalar meson system

For the model of the neutral pseudo-scalar meson system, we adopt the following mass matrix incorporated with the axial U(1) anomaly which was introduced by Pisarski and Wilczek [3] and was able to reproduce the real mass spectrum at zero temperature.

$$
\begin{aligned}
M^2_{\pi^0\pi^0} &= (m_u + m_d)\frac{v}{f_\pi^2}, \\
M^2_{\pi^0\eta} &= \frac{(m_u - m_d)}{\sqrt{3}}\frac{v}{f_\pi^2}, \\
M^2_{\eta\eta} &= \frac{(m_u + m_d + 4m_s)}{3}\frac{v}{f_\pi^2}, \\
M^2_{\pi_0\eta'} &= \sqrt{\frac{2}{3}}(m_u - m_d)\frac{v}{f_\pi f_{\eta'}}, \\
M^2_{\eta\eta'} &= \frac{\sqrt{2}}{3}(m_u + m_d - 2m_s)\frac{v}{f_\pi f_{\eta'}}, \\
M^2_{\eta'\eta'} &= \frac{2}{3}(m_u + m_d + m_s)\frac{v}{f_{\eta'}^2} + K,
\end{aligned}
\tag{2}
$$

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where M^2 is the (mass)^2 matrix for the $\pi_0 - \eta - \eta'$ system, m_i is the quark mass for i-quark (i=up, down and strange), f_j is the decay constant for j-meson (j=π_0, η and η'), v is the strength of the chiral condensate ($-v =< u\bar{u} >=< d\bar{d} >=< s\bar{s} >$) and K is the free parameter representing the effects of the axial U(1) anomaly.

In order to solve this equation at finite temperature, the temperature dependence of all parameters in the above equation should be given. Pisarski and Wilczek have considered only the K effect of the system.[3] Although there have been several model works which suggest the possible change of other parameters like f_π and v at finite temperature, the recent lattice calculation indicates that the temperature dependence of these quantities seems very small up to very close to the critical temperature [7].

So in this paper we also restrict ourselves on the K effect of the system.

Let us first consider the case that the flavor SU(3) symmetry is unbroken, i.e., $m_u = m_d = m_s$. In this case, K is nothing but the mass difference m_0 between singlet and octet meson which is directly related the topological charge by the result of ref.[6]. Then once we know the topological charge distribution $\rho(Q)$, we can estimate K as,

$$K = \bar{m}_0 \equiv \int dQ \tilde{\rho}(Q) m_0(Q), \tag{3}$$

where

$$\tilde{\rho}(Q) = \rho(Q)/\int dQ \rho(Q) \tag{4}$$

is the normalized topological charge distribution.

In eq.2, it is also clear that K represents the strength of the axial

U(1) anomaly which is connected with the gauge field topology via eq.1. So it is rather plausible to assume that the K is independent of the flavor SU(3) symmetry. Under this assumption, above equation becomes valid even in the flavor symmetry broken phase.

Next we assume the singlet-octet mass difference m_0 is purely due to the gauge field topology, i.e., all temperature effect to m_0 should be included through the change of the topological charge distribution $\rho(Q)$. Then we can use eq.3 even at finite temperature.

3 Topological charge distribution at $T \neq 0$

When we evaluate eq.3 at finite temperature on the lattice, there are two restrictions at this stage;

- $m_0(Q)$ was obtained at only one $\beta(= 6/g^2)$ [6],

- unknown non-perturbative behavior of β-function.

In order to get rid off these restrictions, we have decided to make simulation at fixed $\beta(= 6/g^2)$. In this case, $T \neq 0$ can be realized by changing N_t (= temporal extent of the lattice) through $T = 1/N_t a$. (a is the lattice spacing for all directions.)

The simulation is done on $16^3 \times 16$, 8 and 6 lattice in SU(3) gauge at $\beta = 5.89$. These lattices correspond to the temperature $T \simeq 0$, $0.75T_c$ and $0.93T_c$, respectively, by the fact that the critical β on $16^3 \times 6$ lattice is $\beta_c \sim 5.894$ [9] and by using a^{-1} vs. β data by QCDTARO Collaboration [10]. For the SU(3) simulation code in this analysis, we use QCDMPI [8].

After the thermalization of 1000 pseudo-heat-bath sweeps, the measurement is done every 100 sweeps. Number of configurations used are 3500, 1500 and 3200 for $T \simeq 0$, $T = 0.75T_c$ and $T = 0.93T_c$, respectively. To extract the topological charge Q, we adopt the cooling method [11].

The result of $\tilde{\rho}(Q)$ is shown in Fig.1. In this figure, clear temperature dependence of $\tilde{\rho}(Q)$ is seen. The distribution becomes narrower as the temperature gets close to the critical temperature. This feature can be understood as the partial restoration of the axial U(1) symmetry at finite temperature.

The change of the topological charge distribution $\rho(Q)$ at finite temperature should be treated carefully. If we assume the distribution is Gaussian, $\rho(Q) \propto \exp(-\alpha Q^2)$, the constancy of the topological susceptibility in the confinement phase [12] requires,

$$\alpha \propto N_t^{-1} \tag{5}$$

at fixed β.

In this sense, the distribution becomes narrower as the temperature increases (N_t decreases). So this factor (eq.5) should be taken into account for the quantitative study in the next section.

4 Temperature dependence of K and m_η

For the parameterization of $m_0(Q)$, we adopt two different fits of the result in ref.[6]. One is the least square linear fit (fit A);

$$m_0(|Q|) = c_1|Q| + c_2, \tag{6}$$

with $c_1 = 0.08$ and $c_2 = 0.20$ in MeV units [6]. It is surprising that this linear fit seems very well as you can see in ref.[6]. In this fit c_2 is finite. However it is natural for c_2 to be zero at $|Q|=0$ as far as the flavor SU(3) symmetry is unbroken. So we make another fit (fit B);

$$m_0(|Q|) = c_1'|Q| \tag{7}$$

with $c_1' = 0.16$, such that the average mass splitting is almost equal for these two fits.

Since β in this simulation is different from β in [6], we can not fix the absolute scale without the knowledge of the β-function of SU(3) gauge theory. So we concentrate on the mass ratio normalized to the zero temperature value which is independent of this β discrepancy as far as the fits A and B are concerned.

When we calculate $K(T)$ in eq.3, N_t effect in eq.5 should be eliminated. To do this, in this paper, we normalize $K(T)$ by its Gaussian value,

$$K(T) = \frac{K(T)_{\text{lattice}}}{K(T)_{\text{gaussian}}}. \tag{8}$$

Combining $m_0(Q)$ and eq.3, we then obtain the temperature dependence of K parameter. The result is shown in Fig.2. We can see the dramatic decrease of this quantity as the temperature gets close to T_c.

Finally we solve the eigenvalue problem of eq.2 using this $K(T)$. Other parameters are taken from ref.[3]. The result for $m_\eta(T)$ and $m_{\eta'}(T)$ are shown in Fig.3. In these figures, only statistical error coming from the topological charge measurement has been taken into account. Although systematic error coming from above 2 fits

(fit A and fit B) is not so small, mass decrease near T_c is seen for both η and η'. (m_π does not show any noticeable change in this calculation as in ref.[3].) Especially for η', clear mass shift can be seen (in the case of fit B),

$$
\begin{aligned}
\frac{m_{\eta'}(T)}{m_{\eta'}(T=0)} &= 0.86 \pm 0.02 (T = 0.75 T_c) & (9) \\
&= 0.69 \pm 0.01 (T = 0.93 T_c) & (10)
\end{aligned}
$$

This feature can also be understood as the direct reflection of the weakenness of the axial U(1) anomaly. From the present result, if we measure this temperature dependence in the experiment, it could be a possible precursor and/or thermometer of the QGP phase transition.

5 Conclusion

We have analyzed the temperature dependence of the topological charge distributions of the SU(3) gauge theory. Measurement has been performed on the lattice at fixed $\beta(= 6/g^2)$ which enable us to evaluate the temperature dependence of the physical quantity free from the uncertainty of β-function. Although the systematic error coming from the uncertainty of $m_0(Q)$ is not so small, we find the clear mass shift of η' meson near the critical temperature. This is a first clear evidence of the mass shift of η' meson at finite temperature.

There are several assumptions in this paper. Although these assumptions seem plausible at this stage, the detail investigation are highly expected in order to confirm the present result. Calculations

with dynamical quark effect should be performed in future because the dynamical quarks may play a crucial role on this problem.

Acknowledgements

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figure caption

- Figure 1

  Topological charge distributions $\tilde{\rho}(Q)$ at (a) $T \simeq 0$, (b) $T = 0.75T_c$ and (c) $T = 0.93T_c$.

- Figure 2

  $K(T)$ normalized to the zero temperature value. $\bullet$ is for $m_0(|Q|) = c_1|Q| + c_2$ fitting and $\circ$ is for $m_0(|Q|) = c'_1|Q|$ fitting. Errorbars represent only statistical errors.
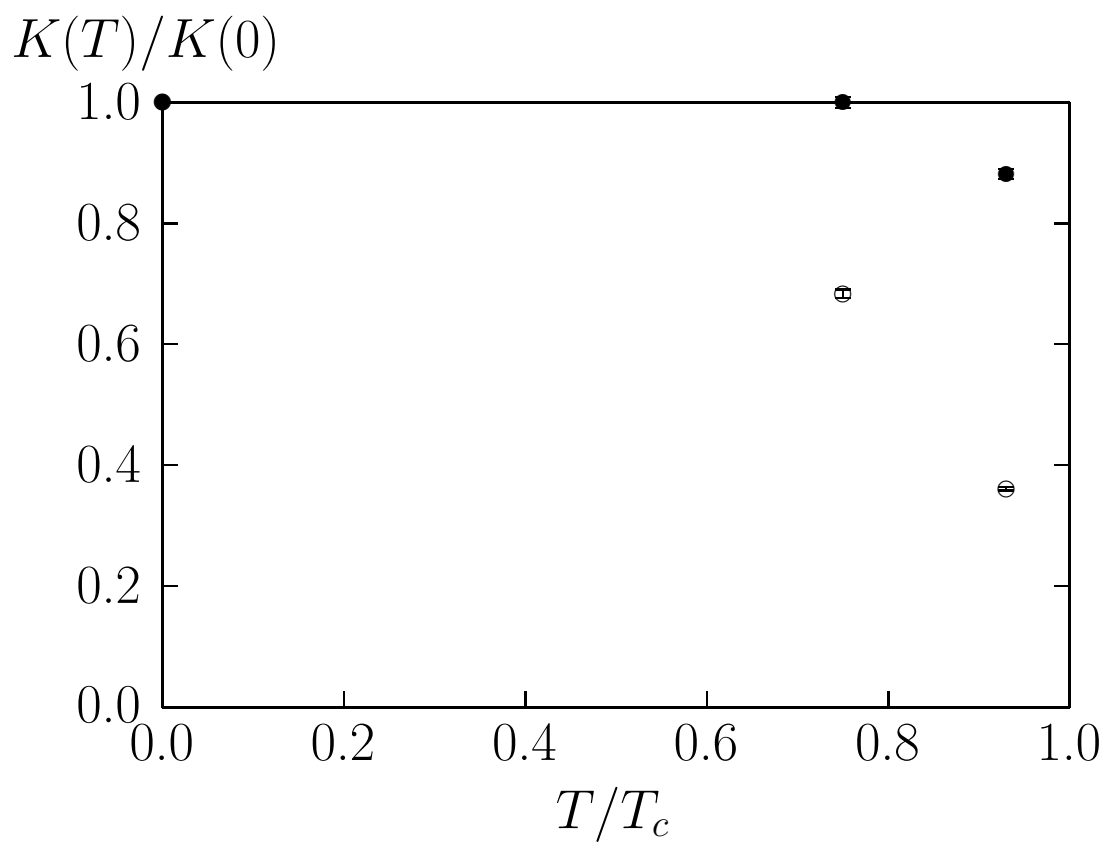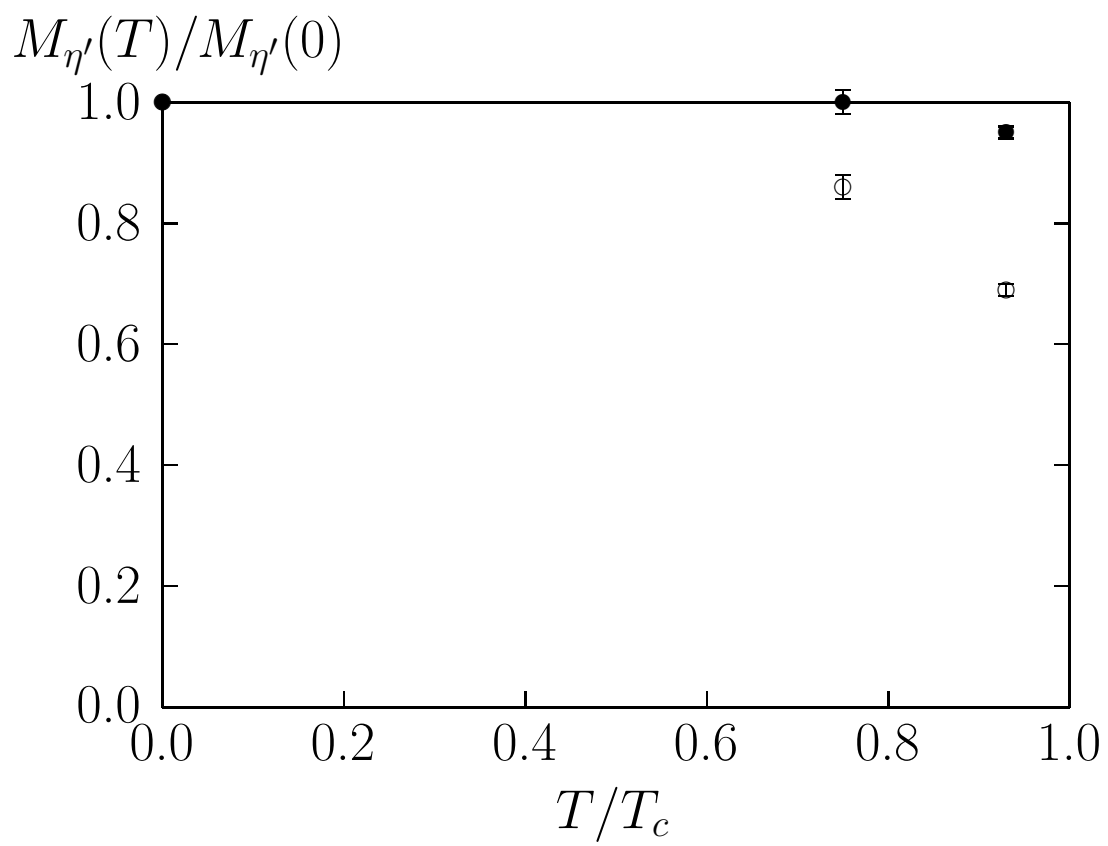
- Figure 3

  (a) η' and (b) η mass vs. temperature normalized to the zero temperature value using the $m_0(|Q|)$ result by Fukugita et al.[6], where m_0 is the η'-octet mass splitting. $\bullet$ is for $m_0(|Q|) = c_1|Q| + c_2$ fitting and $\circ$ is for $m_0(|Q|) = c'_1|Q|$ fitting. Errorbars represent only statistical errors.
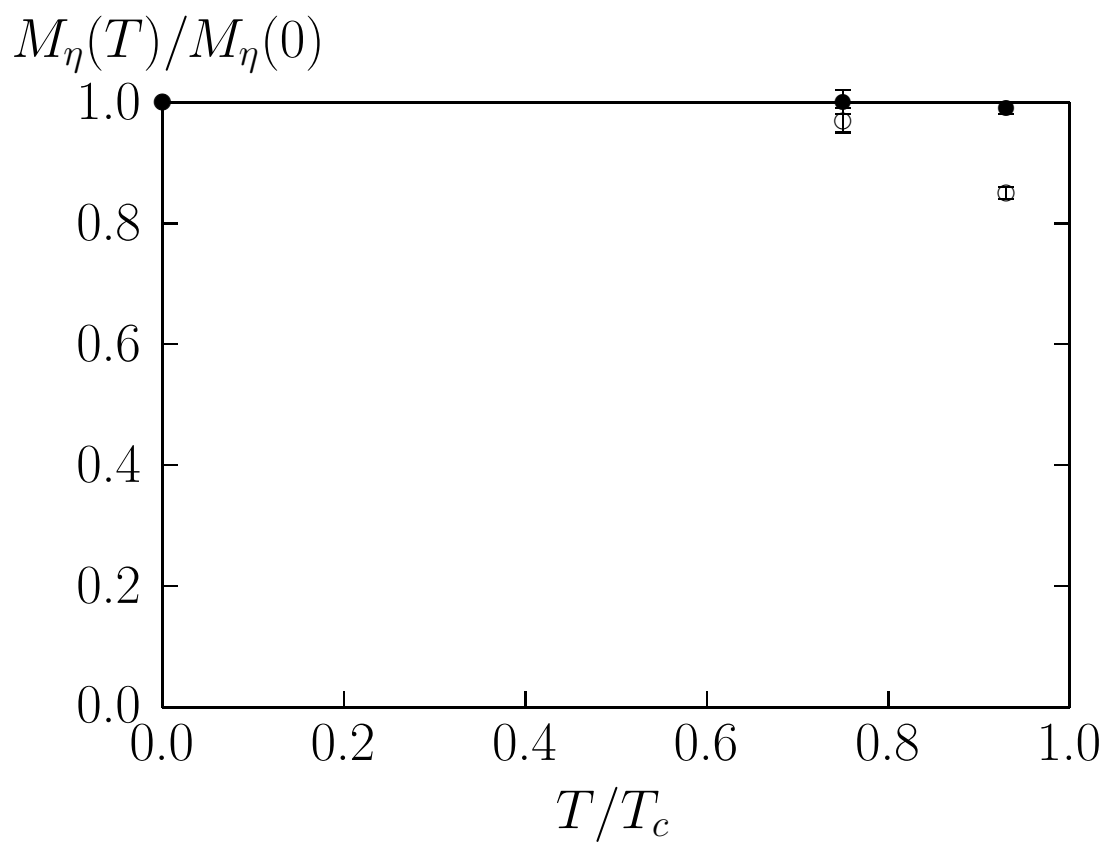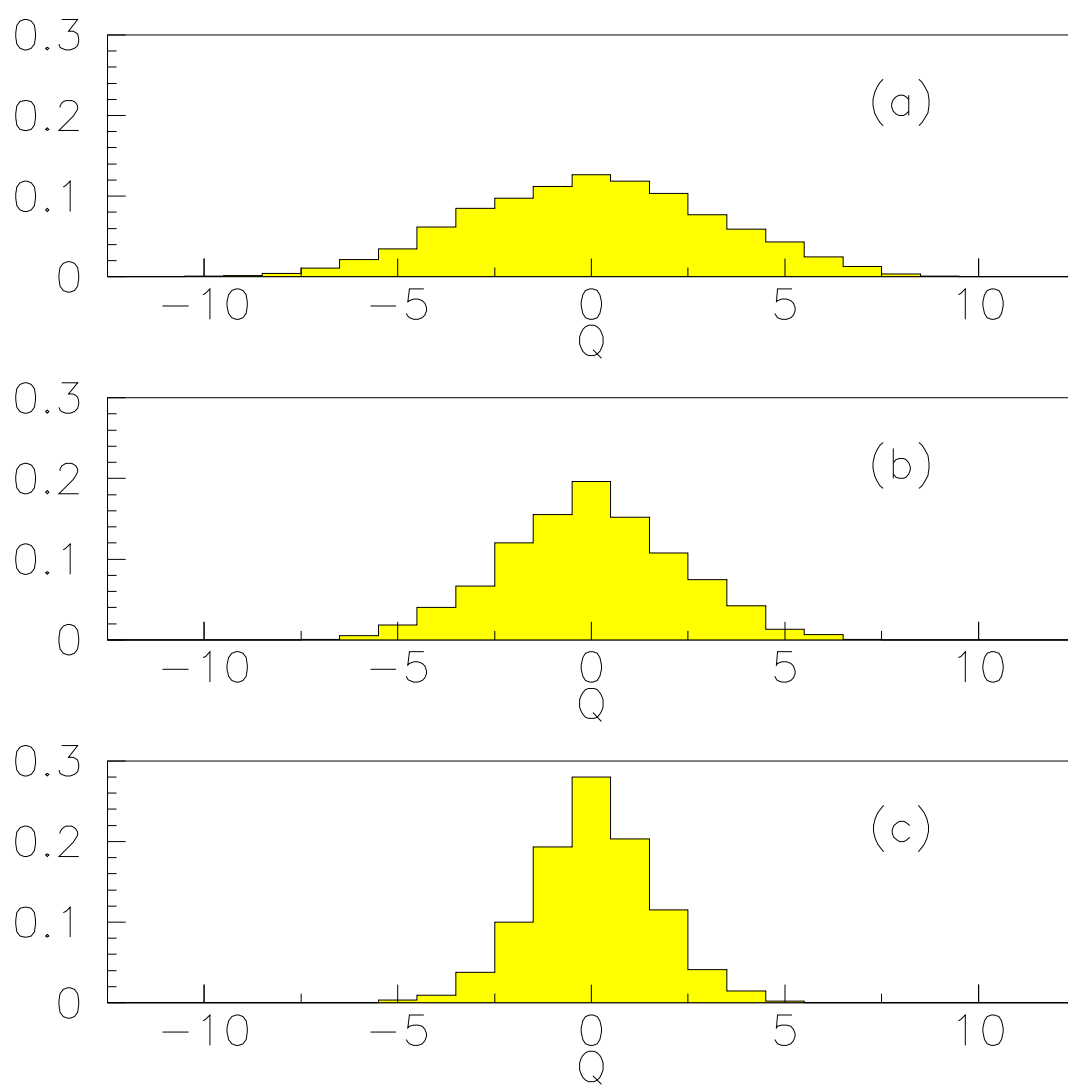
Figure 2

Figure 3(a)

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Figure 3(b)

Figure 1